

# Magnetoelectric Properties of Multiferroic $RMnO_3$

Thomas Lottermoser  
Tsukuba, March 2004

# Magnetoelectric Properties of Multiferroic $RMnO_3$

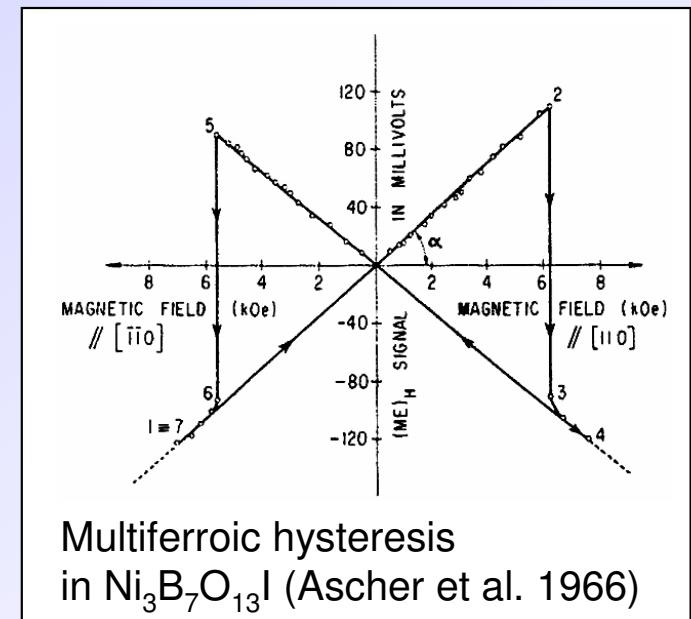
---

- Multiferroics and the Magnetoelectric Effect
- Nonlinear Optics
- Hexagonal Manganites
- Experimental Results
- Summary

# Multiferroic Compounds

Compounds with simultaneous (anti-)ferromagnetic, ferroelectric and/or ferroelastic ordering (Aizu 1969)

⇒ Multiferroics



1958 Idea of new compounds with coexisting magnetic and electric ordering by Smolenskii and Ioffe

1966 First experimental proof of a “multiferroic effect” by Ascher et al.

1975 Suggestions for technical applications based on multiferroic properties by Wood and Austin

...

2000 “Why are there so few magnetic ferroelectrics?” by Hill

# Linear Magnetoelectric Effect

Polarization and magnetization of a medium:

$$P_i = \epsilon_0 \chi_{ij}^e E_j \quad M_i = \chi_{ij}^m H_j$$

Covariant relativistic formulation:

$$\mu_o c M^{\alpha\beta} = \frac{1}{2} \xi_{\mu\nu}^{\alpha\beta} F^{\mu\nu} \quad \text{with:}$$

Relativistic equivalence of electric and magnetic fields requires "**magneto-electric**" cross-correlation ( $\sim \alpha$ ) in matter:

$$P_i = \epsilon_0 \chi_{ij}^e E_j + \frac{1}{c} \alpha_{ij} H_j \quad M_i = \chi_{ij}^m H_j + \frac{1}{\mu_o c} \alpha_{ji} E_j$$

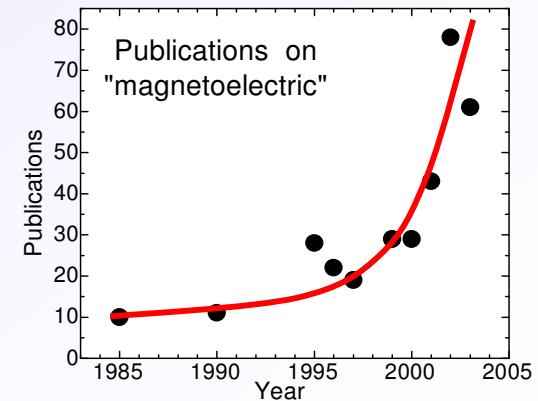
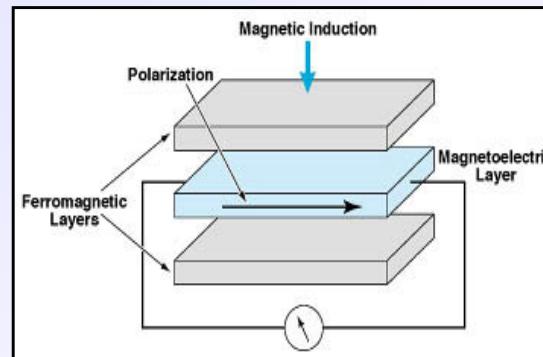
**1960:**

- Small effect ( $10^{-5}$ )
- Limited choice of compounds
- Theoretically not well understood

**2000:**

- New materials with structural (heterostructures) or gigantic (multiferroic) magnetoelectric effects
- New theoretical concepts

$$\left. \begin{array}{l} M_{\alpha\beta} = \begin{pmatrix} 0 & cP_x & cP_y & cP_z \\ -cP_x & 0 & -M_z & M_y \\ -cP_y & M_z & 0 & -M_x \\ -cP_z & -M_y & M_x & 0 \end{pmatrix} \\ F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{pmatrix} \end{array} \right\}$$



# Magnetoelectric Properties of Multiferroic $RMnO_3$

---

- Multiferroics and the Magnetoelectric Effect
- Nonlinear Optics
- Hexagonal Manganites
- Experimental Results
- Summary

# Optical Second Harmonic Generation

In general: Multipole expansion of source term  $\vec{S}$  for SHG:

$$\vec{S} = \mu_0 \frac{\partial^2 \vec{P}^{NL}}{\partial t^2} + \mu_0 \left( \vec{\nabla} \times \frac{\partial \vec{M}^{NL}}{\partial t} \right) - \mu_0 \left( \vec{\nabla} \frac{\partial^2 \hat{Q}^{NL}}{\partial t^2} \right)$$

⇒ Three nonlinear contributions:

Electric dipole (ED):

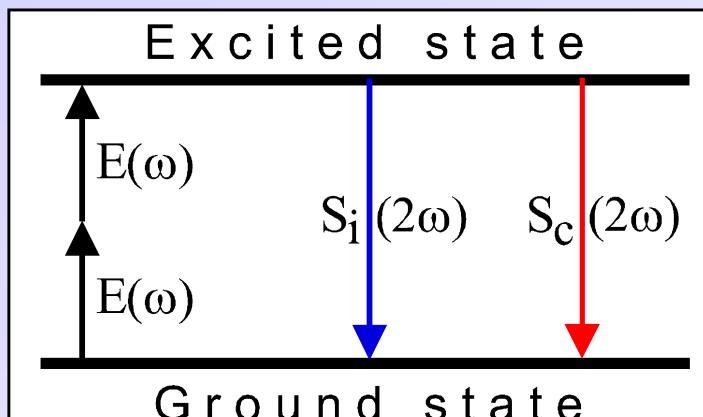
$$\vec{P}^{NL}(2\omega) \propto \hat{\chi}^{ED} : \vec{E}(\omega) \vec{E}(\omega)$$

Magnetic dipole (MD):

$$\vec{M}^{NL}(2\omega) \propto \hat{\chi}^{MD} : \vec{E}(\omega) \vec{E}(\omega)$$

Electric quadrupole (EQ):

$$\hat{Q}^{NL}(2\omega) \propto \hat{\chi}^{EQ} : \vec{E}(\omega) \vec{E}(\omega)$$



Incident  
laser beam

Nonlinear signal:

electric, magnetic,  
i-type  $\propto \chi(i)$  c-type  $\propto \chi(c)$

Interference !

SH source term  $S_i(2\omega) \propto \chi_{ijk} E_j(\omega) E_k(\omega)$

SH intensity:  $I_{SH} \propto |S(c) + S(i)|^2$

$$\propto |\chi(c) + A e^{i\psi} \chi(i)|^2 I^2(\omega)$$

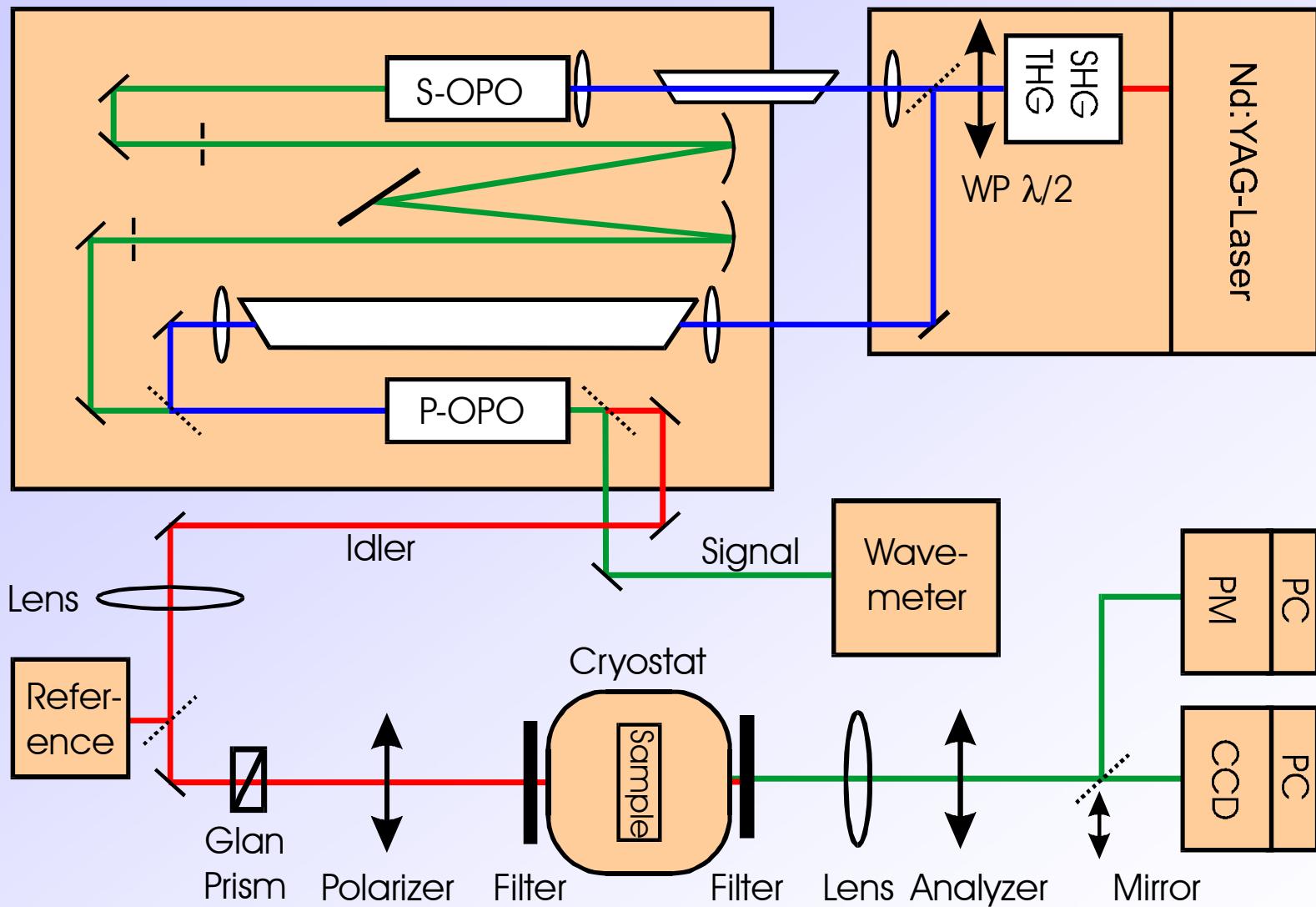
$$= (\chi^2(c) + A^2 \chi^2(i)) + 2A \chi(c) \chi(i) \cos \psi I^2(\omega)$$

always  $> 0$

interference term

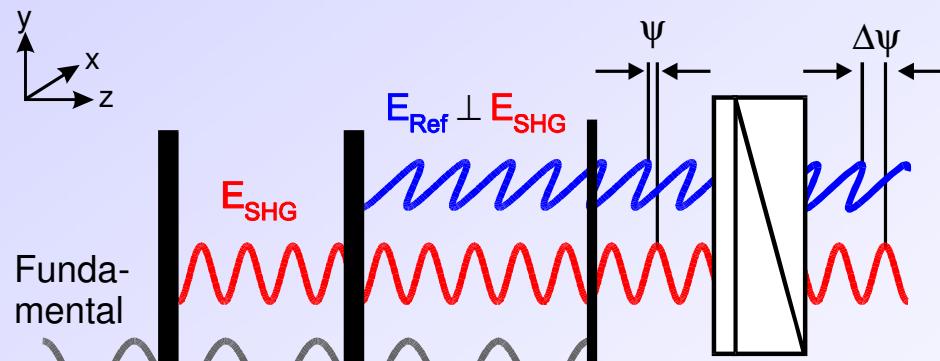
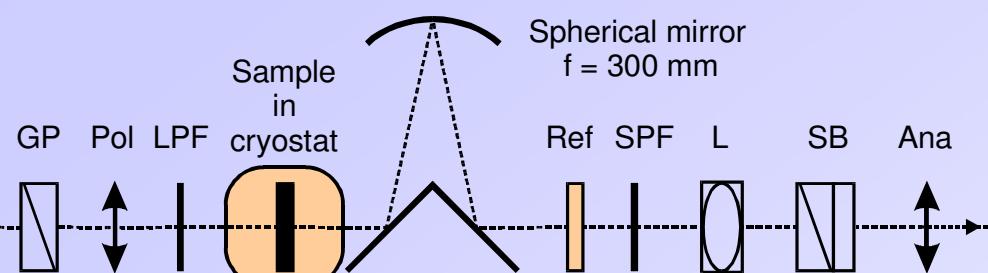
Amplitude A and phase  $\psi$  can be controlled in the experiment.

# Experimental Setup



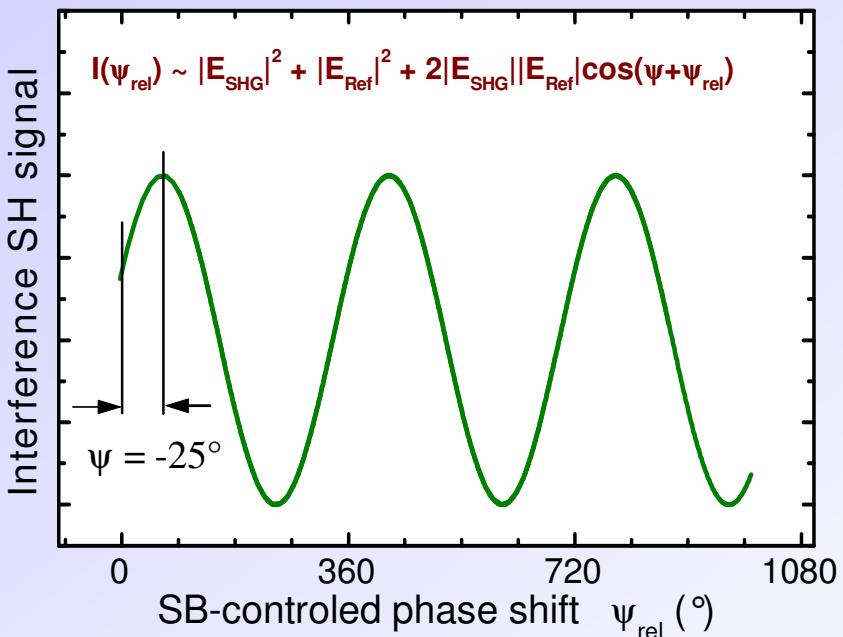
Basic setup with a pulsed Nd:YAG - OPO laser system (3 ns,  $\leq 100$  Hz, 0.4 - 3.0  $\mu\text{m}$ )

# Phase Resolved SH Imaging

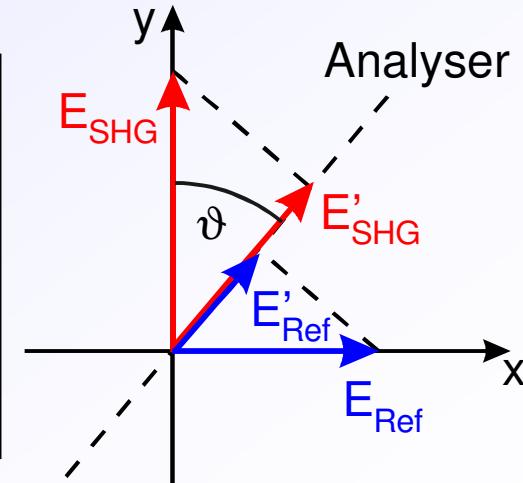
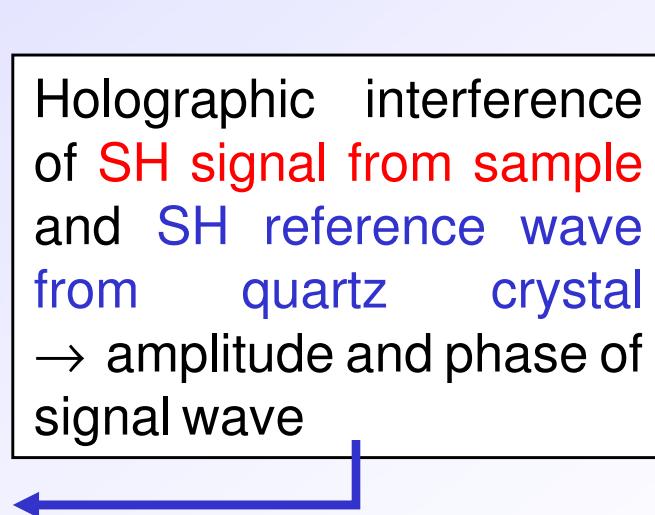


Achromatic beam imaging of sample onto reference crystal in phase measurements

Soleil-Babinet:  
Quartz assembly with tunable birefringence

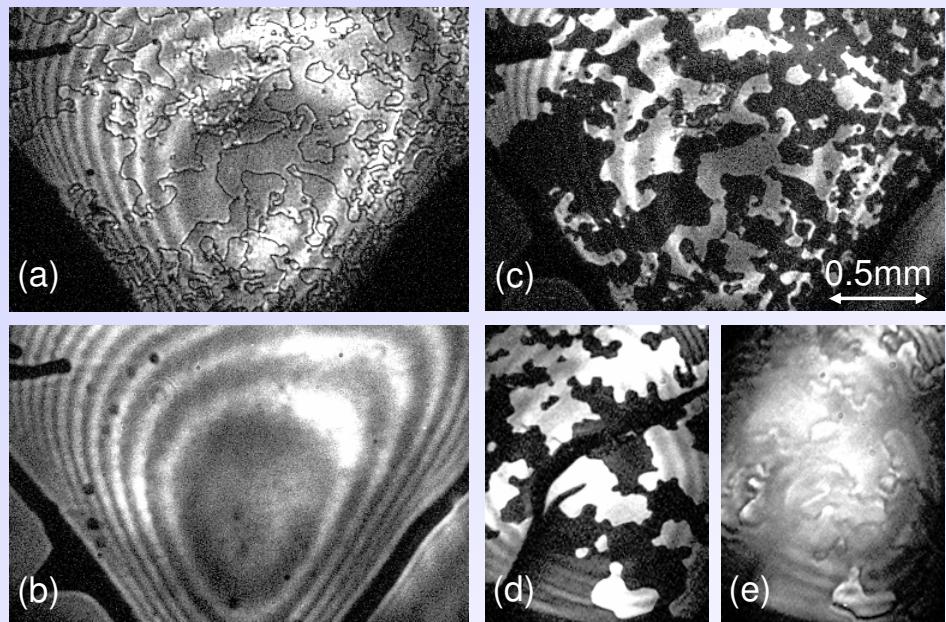
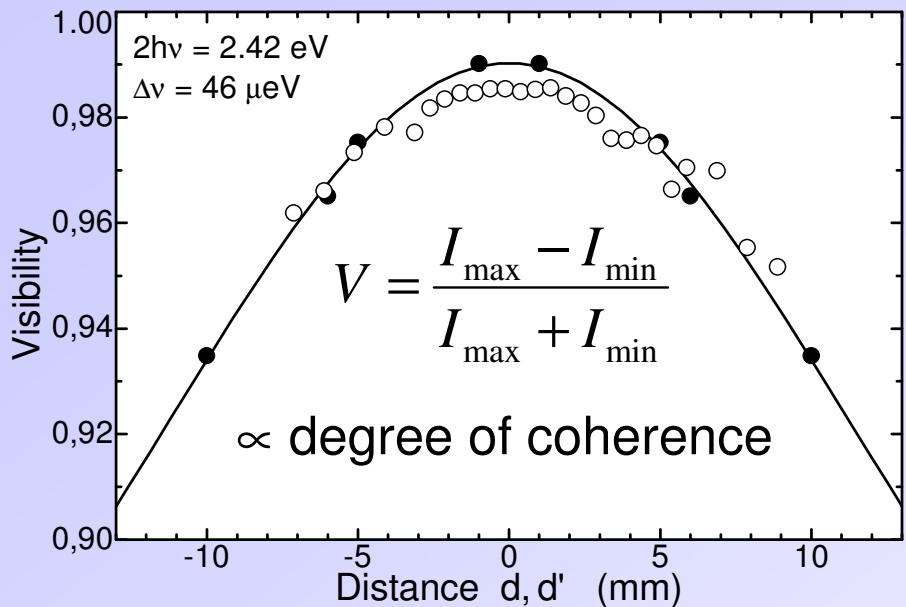


Holographic interference of **SH signal from sample** and **SH reference wave from quartz crystal** → amplitude and phase of signal wave

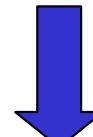


Opt. Lett. **24**, 1520 (1999), Opt. Lett. **29**, 41 (2004)

# Phase-Resolved SH Imaging (Results)



Visibility nearly 100%



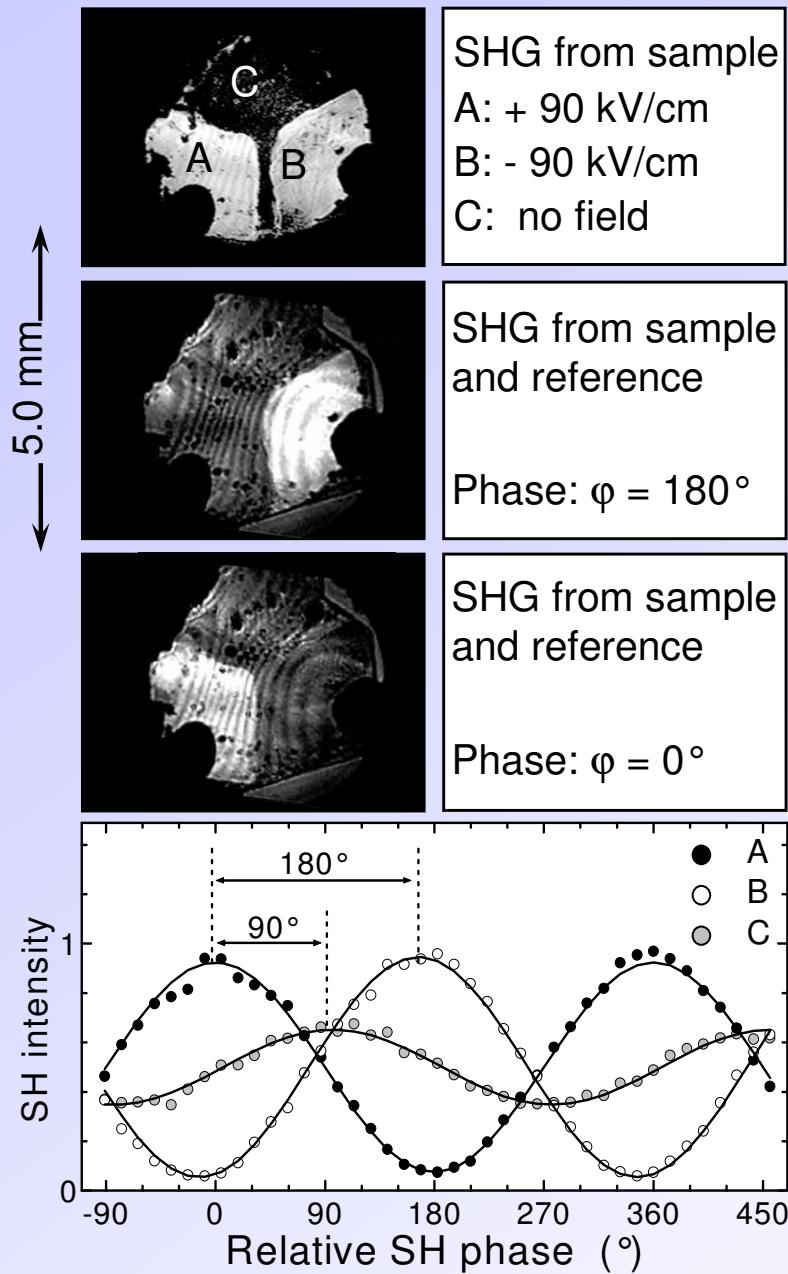
Loss of spatial coherence  
fully compensated!

## Advantages:

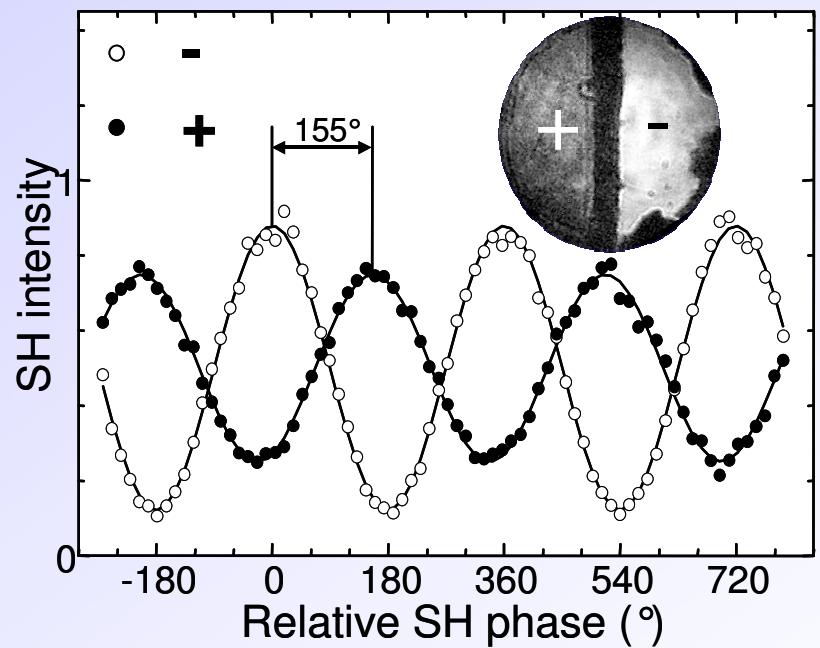
- Allows use of broadband laser sources with poor beam quality
- Large working distances ( $\sim 1 \text{ m}$ )
- More experimental freedom
- Improved image quality

Opt. Lett. **29**, 41 (2004)

# SHG on Ferroelectric HoMnO<sub>3</sub> Domains



But also:



- 180° phase difference between SH contributions from opposite FEL domains
- 90° phase shift and drastic decrease of SH intensity in unpoled region
- Many samples exhibit asymmetry with respect to the direction of the poling field

# Magnetoelectric Properties of Multiferroic $RMnO_3$

---

- Multiferroics and the Magnetoelectric Effect
- Nonlinear Optics
- Hexagonal Manganites
- Experimental Results
- Summary

# Multiferroic Manganites $RMnO_3$

First publications by Yakel and Bertaut on ferroelectric and antiferromagnetic properties in 1963

Dramatic increase of worldwide interest:

Groups in Canada, Germany, Japan, Korea, Netherlands, Russia, Spain, USA,...

Main topics:

- Crystallographic and magnetic structure
- Thin films and ferroelectric properties ( $\rightarrow$  application)
- Multiferroic/magnetoelectric properties

Hexagonal manganites  $RMnO_3$  ( $R = Sc, Y, In, Ho, Er, Tm, Yb, Lu$ )

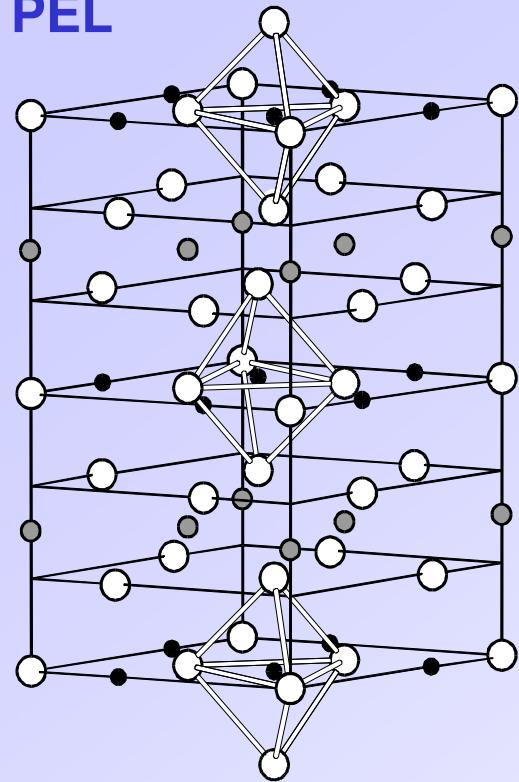
$T < T_C \approx 600\text{-}1000\text{ K}$   $\Rightarrow$  ferroelectric (FEL) + paramagnetic (PM)

$T < T_N \approx 70\text{-}130\text{ K}$   $\Rightarrow$  ferroelectric (FEL) + antiferromagnetic (AFM)

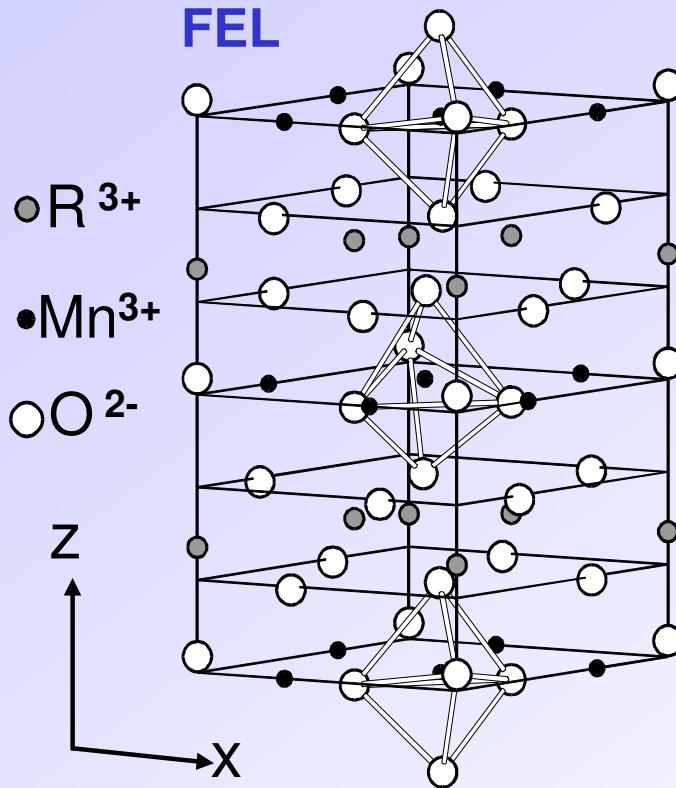
$T < T_{RE} \approx 5\text{ K}$   $\Rightarrow$  FM or AFM order of  $R^{3+}$ -spins for  $R = Ho - Yb$

# Crystallographic and Magnetic Structure

PEL



FEL

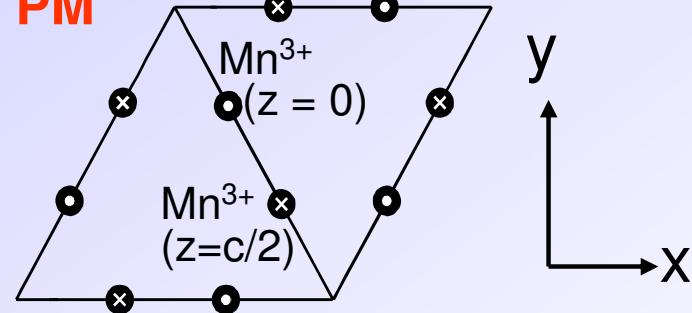


Ferroelectric phase transition:

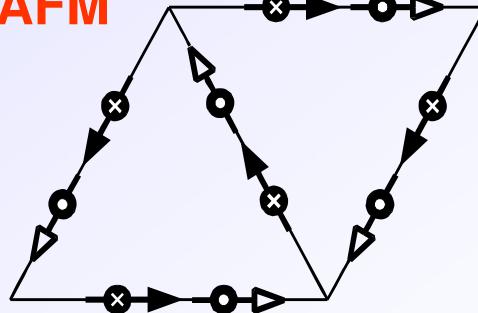
Breaking of inversion symmetry I!

Order parameter:  $\mathcal{P}$

PM



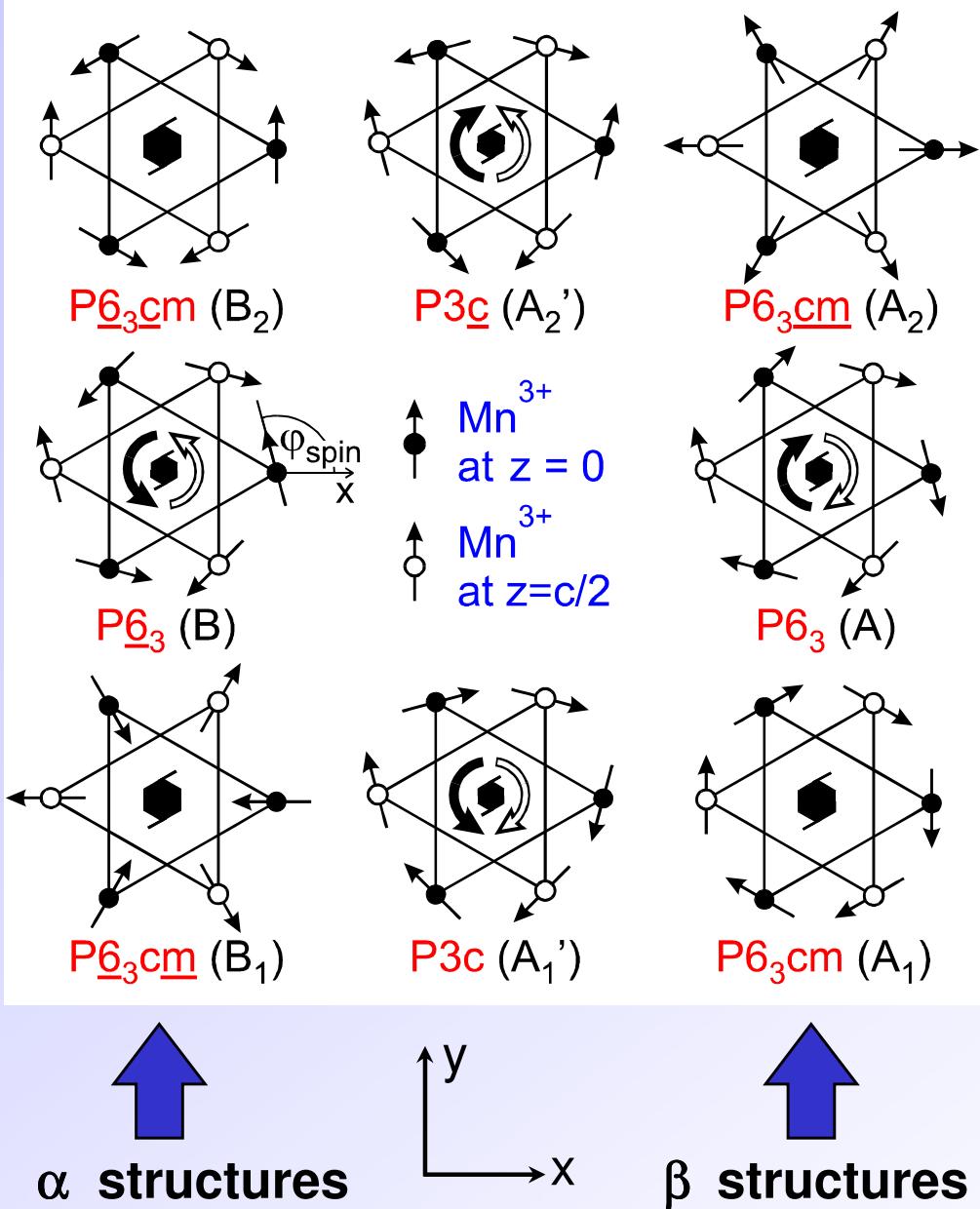
AFM



Antiferromagnetic phase transition of the Mn<sup>3+</sup> sublattice:  
Breaking of time-reversal symmetry T, but *not* of inversion symmetry I!

Order parameter:  $\ell$

# Magnetic Structure and SHG Selection Rules



At least 8 different triangular in-plane spin structures with different magnetic symmetries and different selection rules for SHG

## α structures: SHG for $k||z$ allowed

$$\alpha_x (\varphi = 0^\circ): \quad \chi_{xxx} = 0, \quad \chi_{yyy} \neq 0$$

$$\alpha_y (\varphi = 90^\circ): \quad \chi_{xxx} \neq 0, \quad \chi_{yyy} = 0$$

$$\alpha_p (\varphi = 0-90^\circ): \quad \chi_{xxx} \propto \sin \varphi, \quad \chi_{yyy} \propto \cos \varphi$$

## β structures: SHG for $k||z$ not allowed

$$\beta_x, \beta_y, \beta_p: \quad \chi_{xxx} = 0, \quad \chi_{yyy} = 0$$

## Determine β structure from α-β transition

$$\alpha_x \rightarrow \beta_y: \quad \chi_{xxx} = 0, \quad \chi_{yyy} \propto \cos \varphi$$

$$\alpha_y \rightarrow \beta_x: \quad \chi_{xxx} \propto \sin \varphi, \quad \chi_{yyy} = 0$$

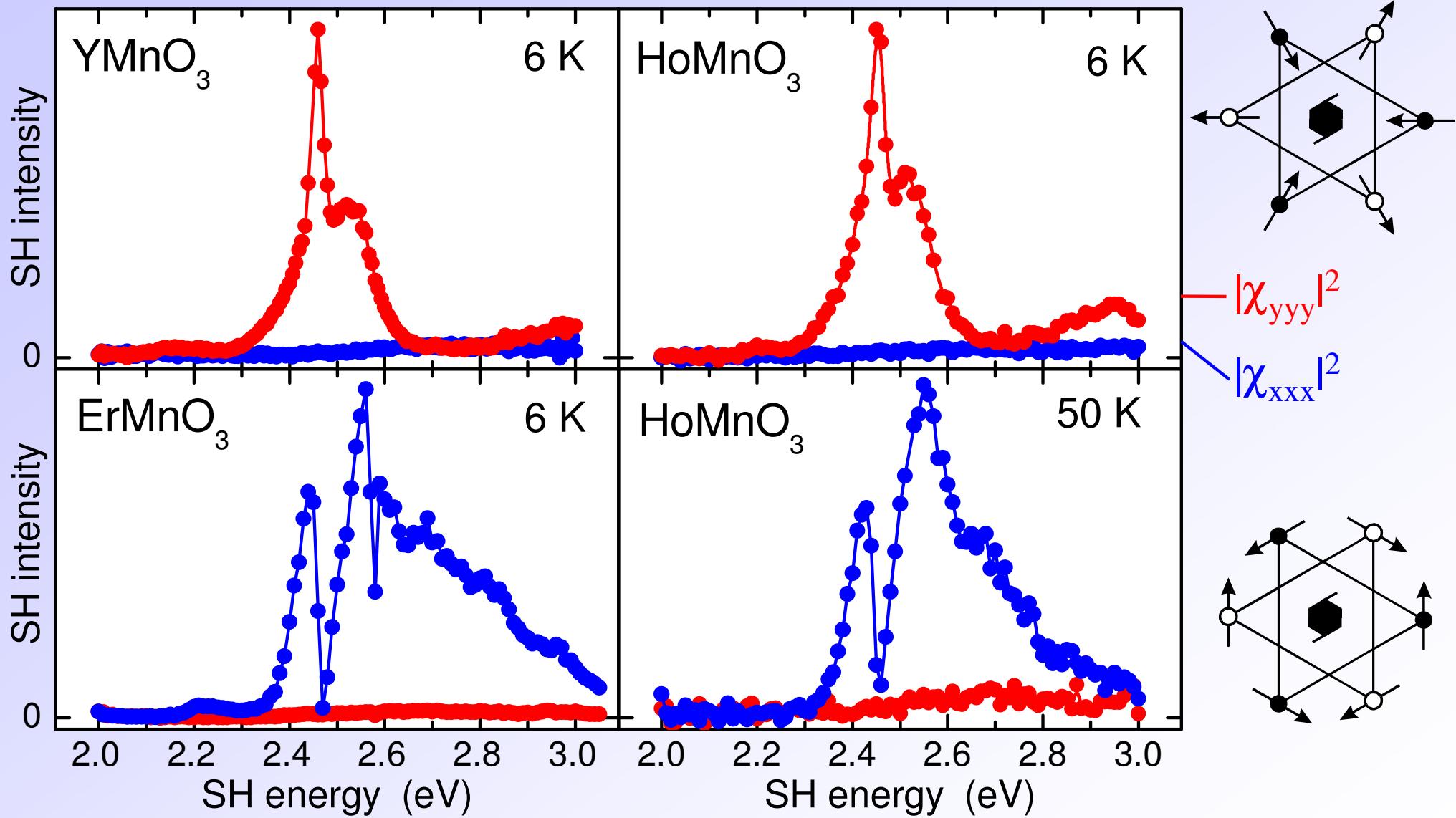
**Contrary to diffraction techniques:  
α and β models clearly distinguishable!**

# Magnetoelectric Properties of Multiferroic $RMnO_3$

---

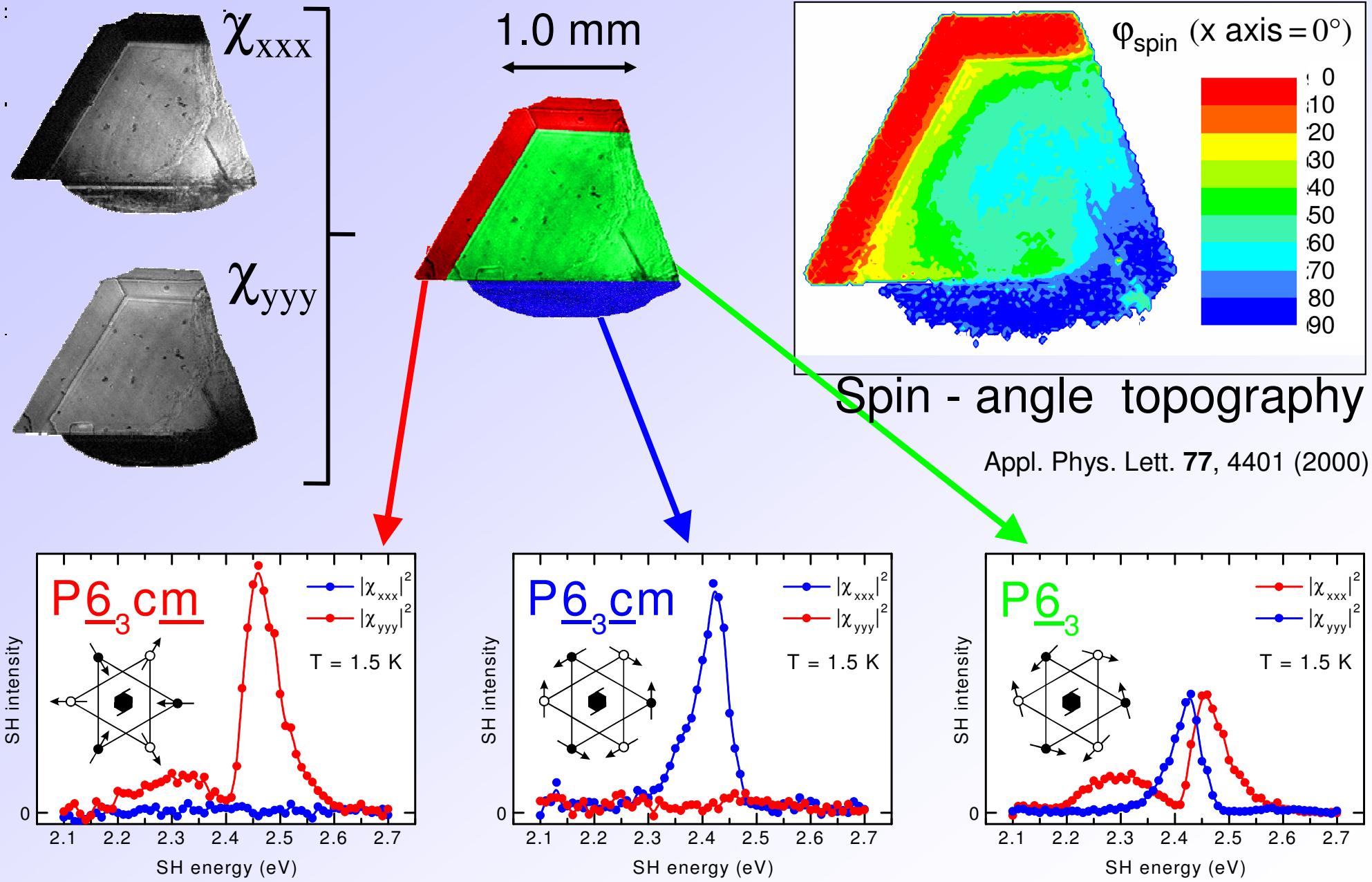
- Multiferroics and the Magnetoelectric Effect
- Nonlinear Optics
- Hexagonal Manganites
- **Experimental Results**
- Summary

# SH spectrum and Magnetic Symmetry



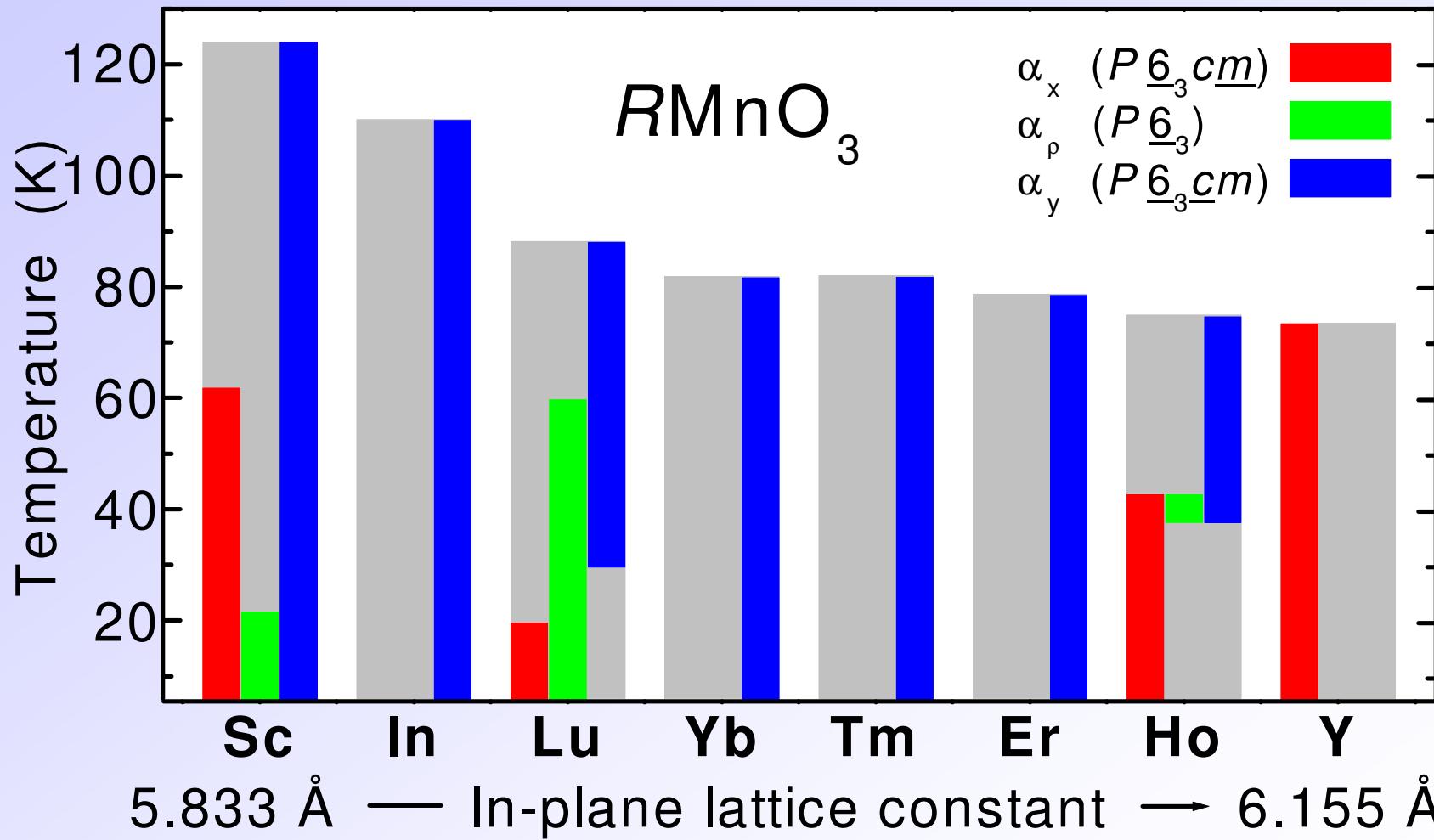
The magnetic symmetry, *not* the R ion, determines the SH spectrum of  $RM\text{nO}_3$

# Phase Coexistence and Spin Topography in $\text{ScMnO}_3$



# Magnetic Symmetry of Hexagonal $RMnO_3$

Second harmonic generation is the only technique capable of the determination of this magnetic phase diagram!



Phys. Rev. Lett. **84**, 5620 (2000)

# SHG in a Multiferroic Compound

Two-dimensional expansion of the SH susceptibility  $\chi$  for electric and magnetic order parameters

$$\vec{P}^{NL}(2\omega) = \epsilon_0 [\hat{\chi}(0) + \hat{\chi}(\wp) + \hat{\chi}(\ell) + \hat{\chi}(\wp\ell) + \dots] \vec{E}(\omega) \vec{E}(\omega)$$

- $\chi(0)$ : Paraelectric paramagnetic contribution — always allowed
  - $\chi(\wp)$ : (Anti)ferroelectric contribution
  - $\chi(\ell)$ : (Anti)ferromagnetic contribution
  - $\chi(\wp\ell)$ : Ferroelectromagnetic contribution
- ] ] allowed below  
the respective  
ordering temperature

- SHG allows simultaneous investigation of magnetic and electric structures
- Selective access to electric and magnetic sublattices
- Ferroelectromagnetic contribution reveals the magneto-electric interaction between the sublattices

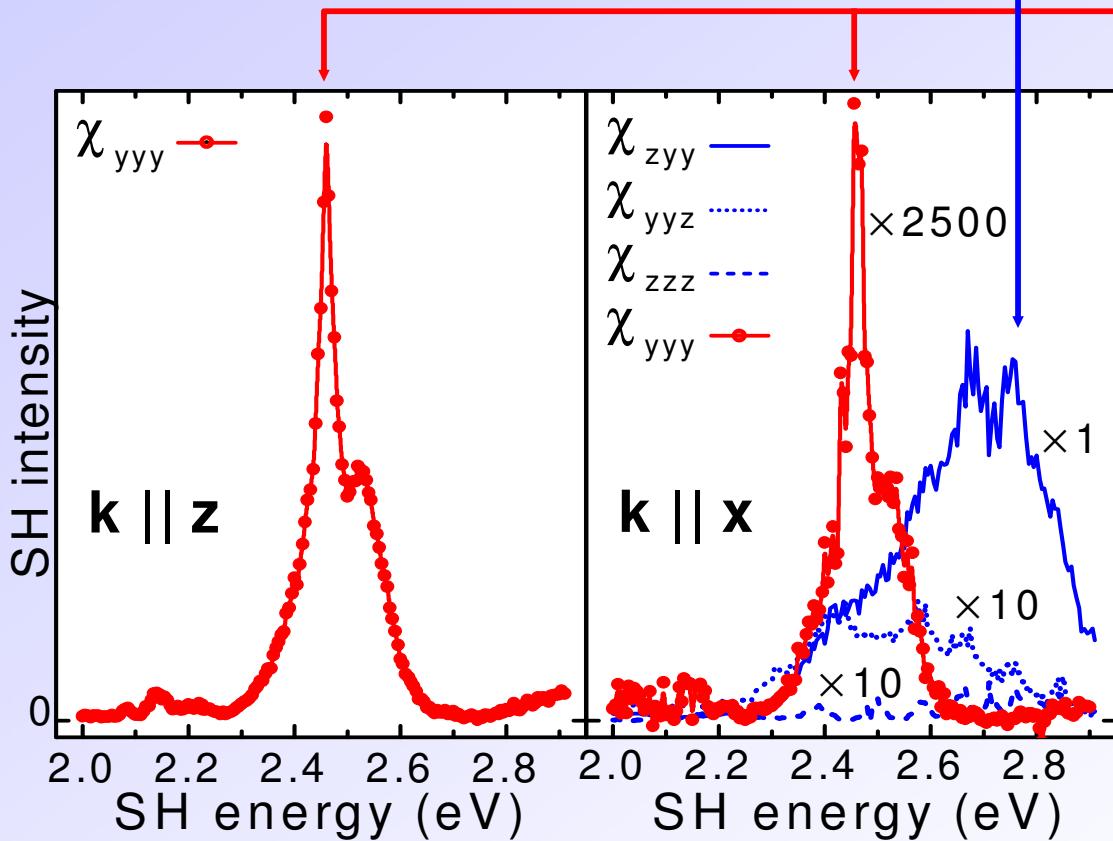
# Symmetry analysis

Ordered Sublattice	Space group	Parity - type symmetry operation	Order parameter
(para)	P6 <sub>3</sub> /mmc	I, T, IT	---
FEL	P6 <sub>3</sub> cm	T	$\mathcal{P}$
AFM	P6 <sub>3</sub> / <u>mcm</u>	I	$\ell$
FEL + AFM	P <u>6</u> <sub>3</sub> <u>cm</u>	---	$\mathcal{P} \cdot \ell$

		$\mathbf{S}^{ED}(\mathcal{P})$	$\mathbf{S}^{MD}(\ell)$	$\mathbf{S}^{EQ}(\ell)$	$\mathbf{S}^{ED}(\mathcal{P} \cdot \ell)$
$\mathbf{k} \parallel \mathbf{x}$	$\mathbf{S}_y$	$2i_1 E_y E_z$	---	---	$e_1 E_y^2$
	$\mathbf{S}_z$	$i_2 E_y^2 + i_3 E_z^2$	---	---	---
$\mathbf{k} \parallel \mathbf{y}$	$\mathbf{S}_x$	$2i_1 E_x E_z$	---	$-2q_1 E_x E_z$	---
	$\mathbf{S}_z$	$i_2 E_x^2 + i_3 E_z^2$	$m_1 E_x^2$	$-q_2 E_x^2$	---
$\mathbf{k} \parallel \mathbf{z}$	$\mathbf{S}_x$	---	$-2m_1 E_x E_y$	$-2q_3 E_x E_y$	$-2e_1 E_x E_y$
	$\mathbf{S}_y$	---	$m_1(E_y^2 - E_x^2)$	$q_3(E_y^2 - E_x^2)$	$e_1(E_y^2 - E_x^2)$

# Magnetoelectric Second Harmonic Generation

Source term	$S^{\text{ED}}(0)$	$S^{\text{ED}}(\mathcal{P})$	$S^{\text{MD,EQ}}(\ell)$	$S^{\text{ED}}(\mathcal{P}\ell)$
Sublattice sym.	P6 <sub>3</sub> /mcm	P6 <sub>3</sub> cm	P6 <sub>3</sub> /mcm	P6 <sub>3</sub> cm
SHG for $k \parallel z$	= 0	= 0	$\neq 0$	$\neq 0$
SHG for $k \parallel x$	= 0	$\neq 0$	= 0	

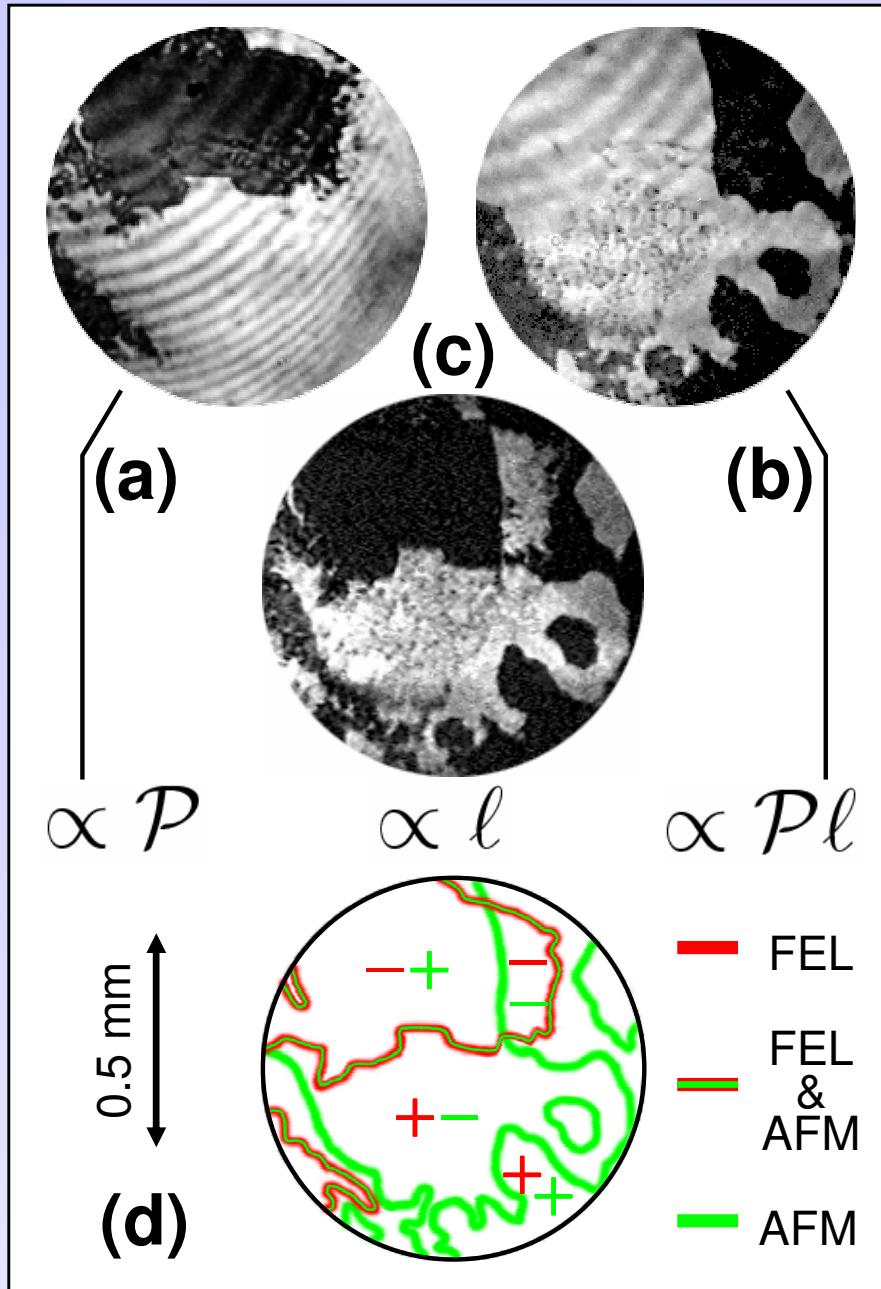


Identical **magnetic** spectra for  $k \parallel z$  and  $k \parallel x$  indicate **bilinear coupling** to  $\mathcal{P}, \ell$ .

Unarbitrary evidence for the first observation of "**magnetoelectric SHG**"

Nature 419, 818 (2002)

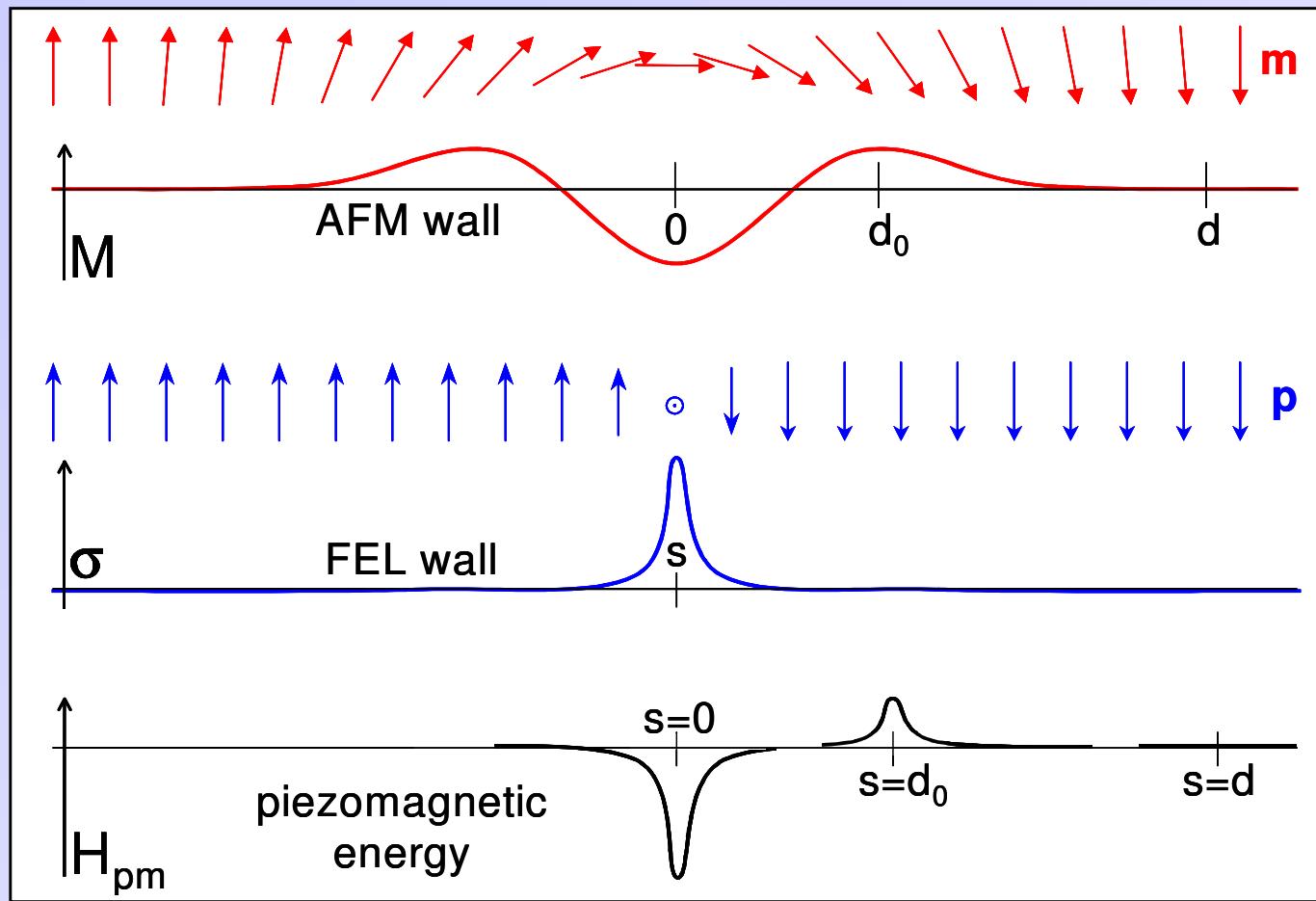
# Observation of Ferroelectromagnetic Domains



- Independent **ferroelectric** ( $\propto P$ ) and **ferroelectromagnetic** ( $\propto Pl$ ) domain structures; **antiferromagnetic** domain structure ( $\propto \ell$ ) is not!
- "Ferroelectromagnetic domains":  
 $Pl = +1$  for  $P = \pm 1, \ell = \pm 1$   
 $Pl = -1$  for  $P = \pm 1, \ell = \mp 1$
- Any reversal of the **FEL** order parameter is clamped to a reversal of the **AFM** order parameter
- Origin: Piezomagnetic interaction between lattice distortions at the **FEL** domain wall and magnetization induced by the **AFM** domain wall decreases the free energy

Nature 419, 818 (2002)

# Interaction of electric and magnetic domain walls



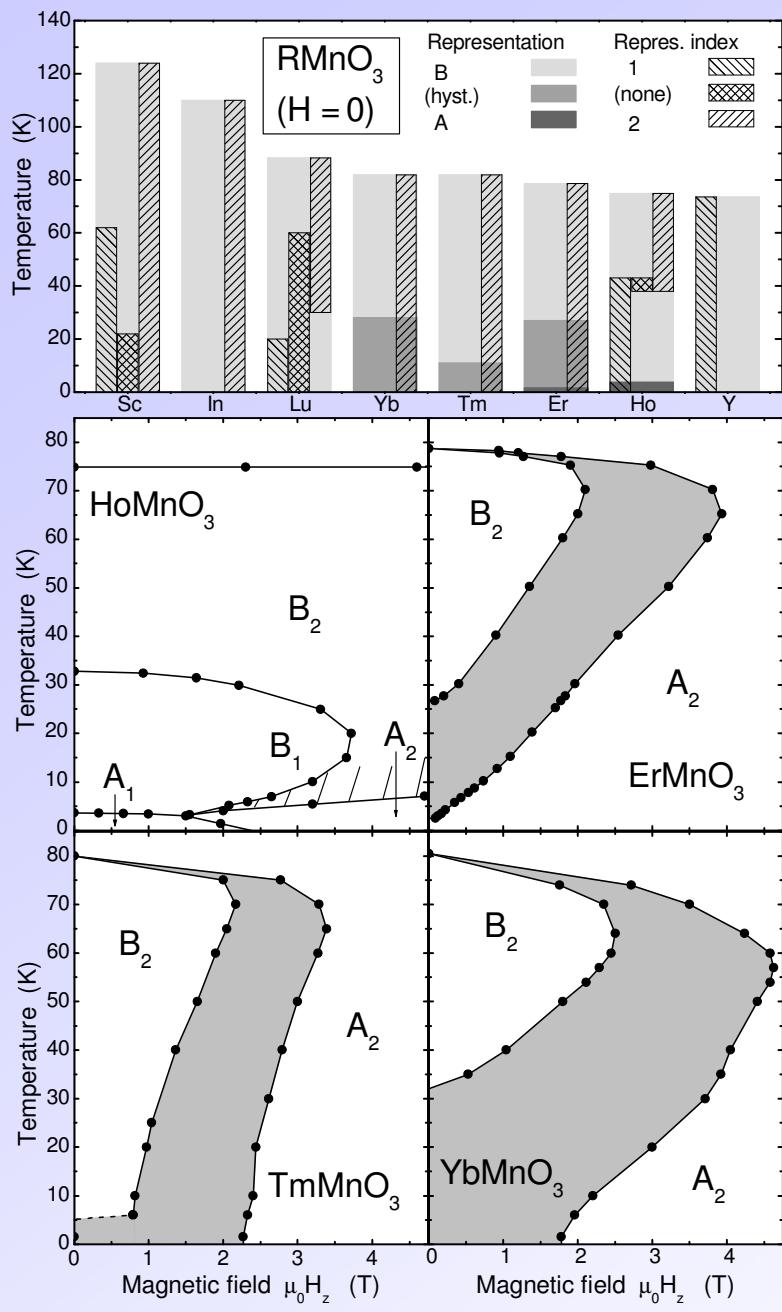
- AFM wall carries an intrinsic macroscopic magnetization
- FEL wall induces strain due to switching of polarization
- Width of walls:
  - AFM - O[10<sup>3</sup>] unit cells: small in-plane anisotropy
  - FEL – O[10<sup>0</sup>] unit cells: large uniaxial anisotropy

Piezomagnetic contribution  $H_{pm} = q_{ijk} M_i \sigma_{jk}$  with  $\sigma \propto P_z$   $\rightarrow$  higher-order magnetoelectric effect

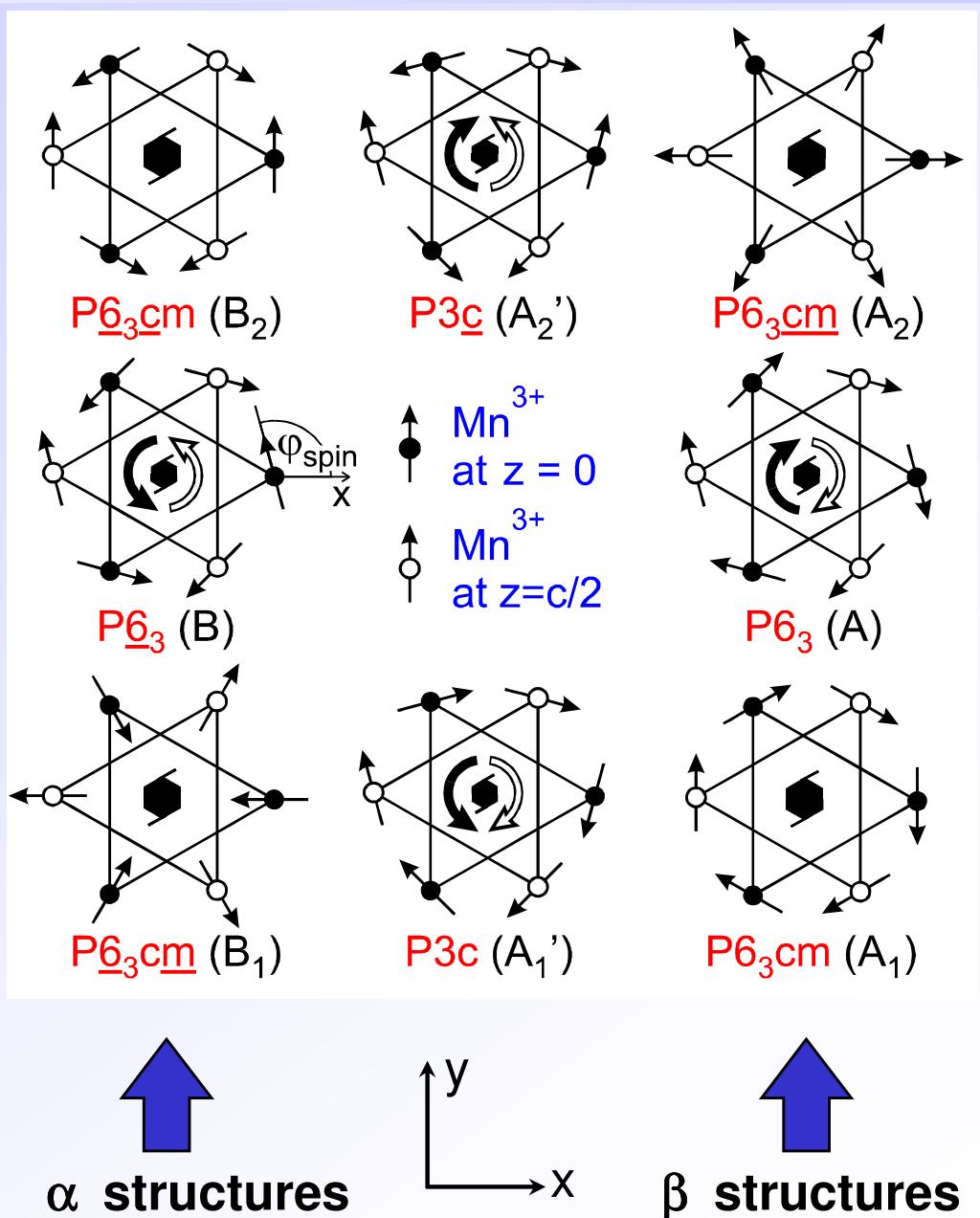
Generation of an antiferromagnetic wall clamped to a ferroelectric wall leads to reduction of free energy.

Phys. Rev. Lett. 90, 177204 (2003)

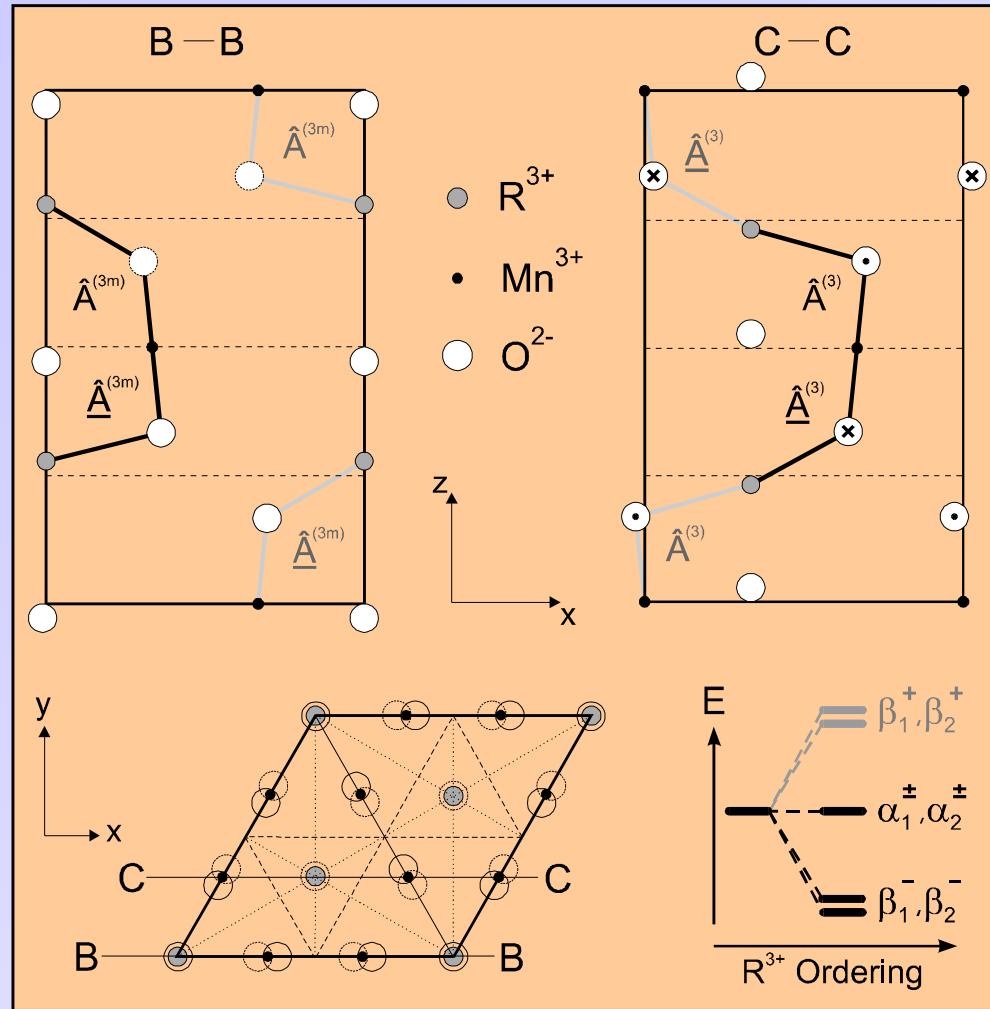
# H/T Phase Diagram of Hexagonal $RMnO_3$



Magnetoelectric Properties of Multiferroic  $RMnO_3$



# Magnetoelectric 3d- 4f Superexchange in RMnO<sub>3</sub>



Gigantic magnetoelectric effect which originates in 3d-4f superexchange; triggers hidden phase transition!

$$H_{\text{ex}} = \sum_{k=3m,3} \sum_{i_k=1}^{4(k=3)} \sum_{j=1}^6 \vec{S}^{R^k(i_k)} \hat{A}^{k,i_k,j} \vec{S}^{\text{Mn}(j)}$$

k: R sites with 3 and 3m symmetries

i<sub>k</sub>: all R ions at k sites (4+2)

j: 6 Mn ions neighboring an R ion

A: Mn-R exchange matrix (4 types)

S: spins of Mn and R ions

**α model:**  $H_{\text{ex}}(\alpha) = 0$  no change!

**β model:** lowers ground-state energy:

$$H_{\text{ex}}^\ell(\beta_x) = 6\ell S^R S^{\text{Mn}} [(A_{zx}^{3m} - \underline{A}_{zx}^{3m}) - (A_{zx}^3 - \underline{A}_{zx}^3)]$$

$$H_{\text{ex}}^\ell(\beta_y) = 6\ell S^R S^{\text{Mn}} [(A_{zy}^{3m} + \underline{A}_{zy}^{3m}) - (A_{zy}^3 + \underline{A}_{zy}^3)]$$

Ferroelectric distortion modifies the superexchange:

$$\delta \hat{A} \equiv \hat{A} - \underline{A}, \quad \delta \hat{A} = \delta \hat{A}_0 P_z \quad \text{Scales with order par.}$$

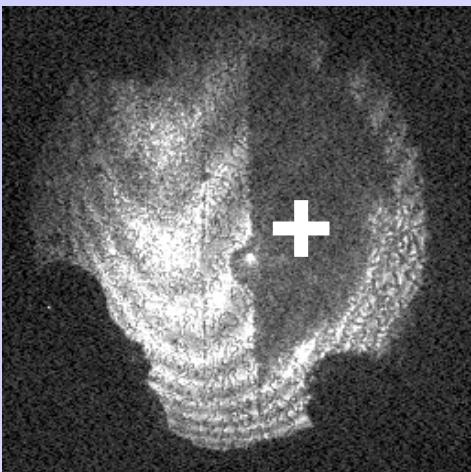
$$\alpha_{zz} \equiv 6\ell S_y^{\text{Mn}} (\delta A_0^{3m} \pm \delta A_0^3)_{zy} \quad \text{Substitution leads to:}$$

$$H_{\text{ex}}(\beta_x) = \alpha_{zz} P_z S_z^R$$

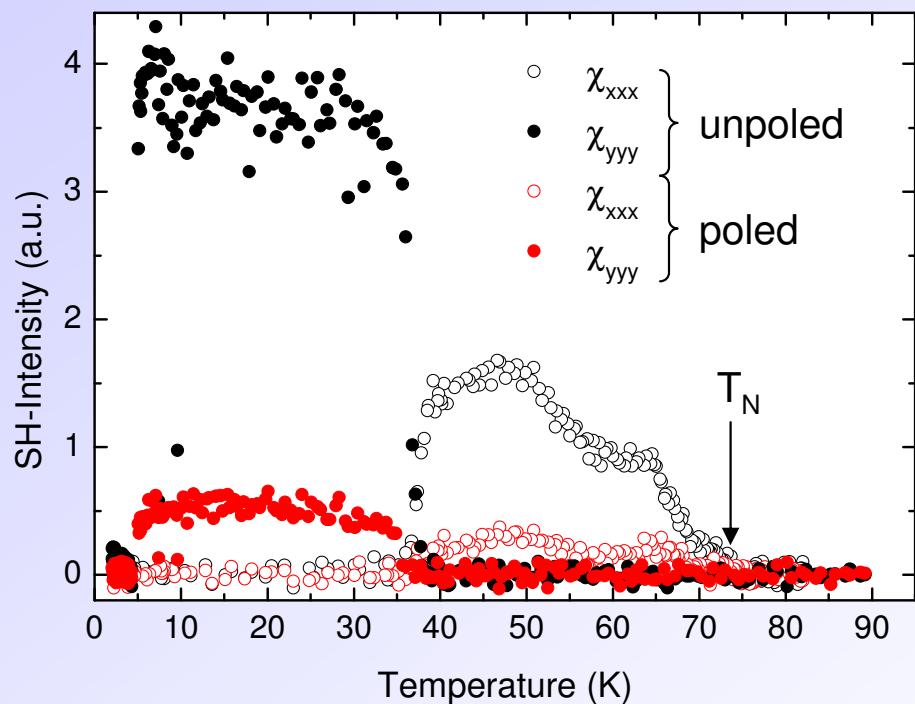
**ME contribution**

# Spontaneous Magnetoelectric Effect in HoMnO<sub>3</sub>

Antiferromagnetic SH



Ferroelectric poling  
quenches magnetic signal!



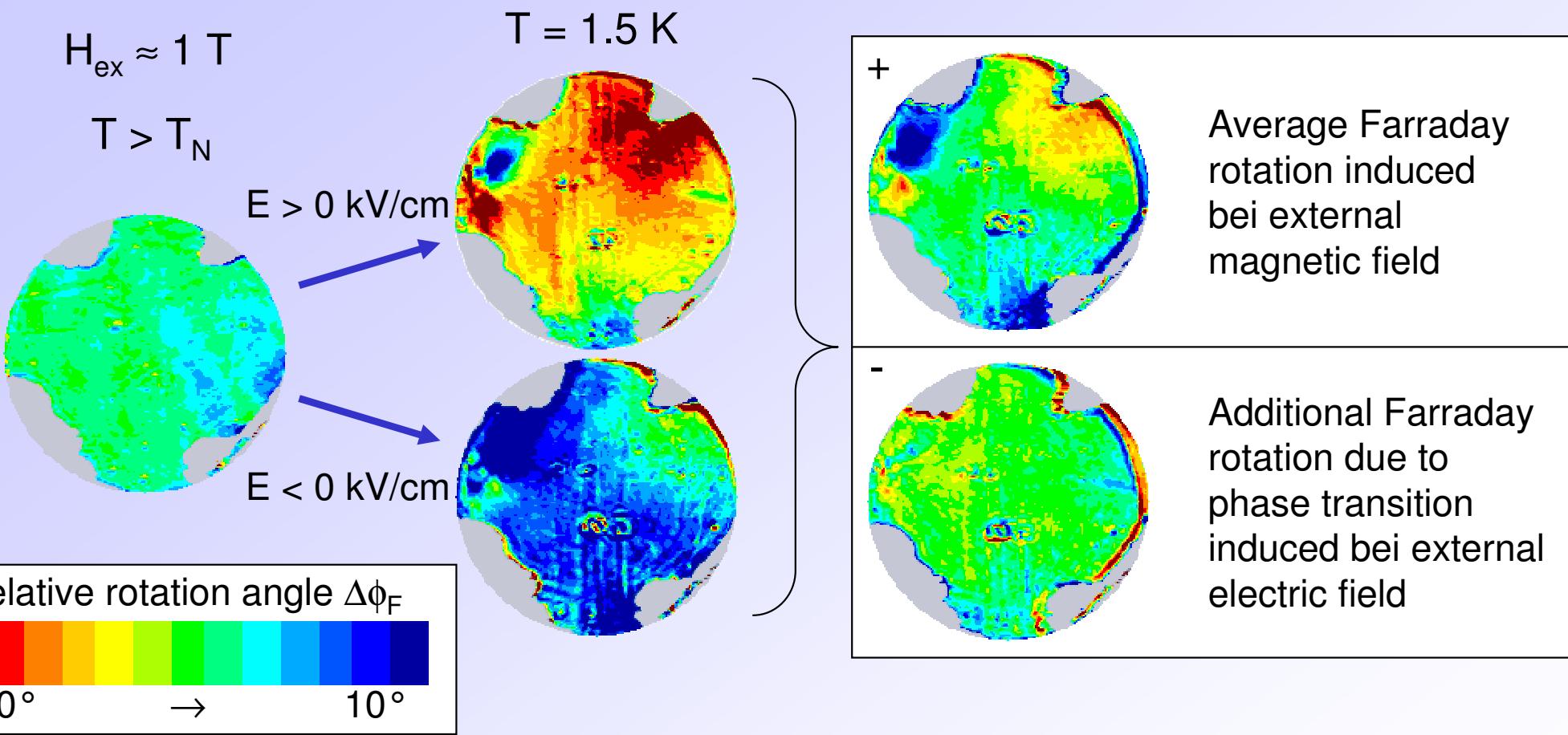
**Only Explanation:**

Magnetic phase transition triggered by the internal electric field!

⇒ **spontaneous magnetoelectric effect!**

Magnetoelectric effect only allowed for  $\beta_x$  phase with ferromagnetic ordering of Ho<sup>3+</sup>-spins!

# Magnetization Control by Electric Field in HoMnO<sub>3</sub>



Farraday rotation depends on the direction of the external **electric** field!

Only possible due to magnetoelectric effect!

Magnetoelectric effect only allowed for  $\beta_x$  phase in HoMnO<sub>3</sub>!

**Evidence of magnetic phase transition induced by magnetoelectric effect!**

# Magnetoelectric Properties of Multiferroic $RMnO_3$

---

- Multiferroics and the Magnetoelectric Effect
- Nonlinear Optics
- Hexagonal Manganites
- Experimental Results
- Summary

# Summary

Multiferroic hexagonal manganites  $RMnO_3$  are a model substance to investigate magnetoelectric interactions in a ferroelectromagnetic material:

- Coupling of ferroelectric and antiferromagnetic order parameters leads to “magnetoelectric” SHG
- “Ferroelectromagnetic” domains due to interaction of ferroelectric and antiferromagnetic domain walls
- Magnetoelectric effect leads to spontaneous phase transition in compounds with  $R = Ho - Yb$
- Control of the magnetic phase by the electric field due to magnetoelectric effect

# Acknowledgment

---

## **Germany:**

D. Fröhlich, M. Fiebig, St. Leute, C. Degenhard, M. Maat, S. Kallenbach, Th. Lonkai

## **Russia:**

R.V. Pisarev, V.V. Pavlov, A.V. Goltsev

## **Japan:**

K. Kohn, Y. Tanabe, E. Hanamura