

Symmetry and coupling of magnetic and electric order parameters in YMnO₃

Th. Lottermoser, M. Fiebig, D. Fröhlich

Institut für Physik, Universität Dortmund, 44221 Dortmund, Germany



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Ferroelectromagnetic RMnO₃

Ferroelectromagnetism:

Simultaneous electric and magnetic ordering

Hexagonal manganites RMnO₃ (R = Sc, Y, Ho, Er, Tm, Yb, Lu)

$T < T_C \approx 600-1000$ K \Rightarrow ferroelectric (FE)
+ paramagnetic (PM)

$T < T_N \approx 70-130$ K \Rightarrow ferroelectric (FE)
+ antiferromagnetic (AFM)

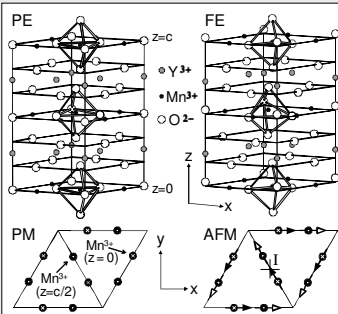
Question: Nature of the coupling between the ferroelectric and antiferromagnetic order?

Experimental method: Second harmonic generation (SHG)

Polarization dependent spectroscopy

Nonlinear phase sensitive domain topography

Electric and magnetic ordering of YMnO₃



Ferroelectric phase transition:

Breaking of inversion symmetry I !

Order parameter: P

Antiferromagnetic phase transition of the Mn³⁺ sublattice:

Breaking of time-reversal symmetry T , but not of inversion symmetry I !

Order parameter: ℓ

Optical second harmonic generation (SHG)

In general: Multipole expansion of source term \vec{S} for SHG:

$$\vec{S} = \mu_0 \frac{\partial^2 \vec{P}^{NL}}{\partial t^2} + \mu_0 \left(\vec{\nabla} \times \frac{\partial \vec{M}^{NL}}{\partial t} \right) - \mu_0 \left(\vec{\nabla} \cdot \frac{\partial^2 \vec{Q}^{NL}}{\partial t^2} \right)$$

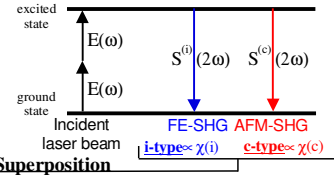
\Rightarrow Three nonlinear contributions:

Electric dipole (ED): $\vec{P}^{NL}(2\omega) \propto \chi^{ED} : \vec{E}(\omega)\vec{E}(\omega)$

Magnetic dipole (MD): $\vec{M}^{NL}(2\omega) \propto \chi^{MD} : \vec{E}(\omega)\vec{E}(\omega)$

Electric quadrupole (EQ): $\vec{Q}^{NL}(2\omega) \propto \chi^{EQ} : \vec{E}(\omega)\vec{E}(\omega)$

SHG of electric-dipole type:



Source term for SHG: $P_i(2\omega) = \epsilon_0 \chi_{ijk}^{SH} E_j(\omega) E_k(\omega)$

Intensity of SH signal: $I_{SH} \propto |P(c) + P(i)|^2$

$$\propto |\chi(c) + A e^{i\varphi} \chi(i)|^2 I^2(\omega)$$

$$= (\chi^2(c) + A^2 \chi^2(i) + 2A \chi(c) \chi(i) \cos \varphi) I^2(\omega)$$

always > 0 interference term

A: amplitude ratio of i-type and c-type terms

φ : phase shift between complex contributions

A and φ can be fully controlled in experiment

Susceptibility χ couples linearly to symmetry!

Order parameter $O \Rightarrow \chi \equiv \chi(O) \propto O$

Observation of ferroelectromagnetic SHG

Order	Space group	Symmetry operation	Order parameter
PE + PM	P6 ₃ /mmc	I, T, IT	---
FE + PM	P6 ₃ cm	T	P
PE + AFM	P6 ₃ /mcm	I	ℓ
FE + AFM	P6 ₃ cm	---	$P \cdot \ell$

Expansion of coupling of SHG to the order parameters:

$$\vec{P}^{NL}(2\omega) = \epsilon_0 (\chi^{ED}(0) + \chi^{ED}(P) + \chi^{ED}(\ell) + \chi^{ED}(P\ell)) \vec{E}(\omega)\vec{E}(\omega)$$

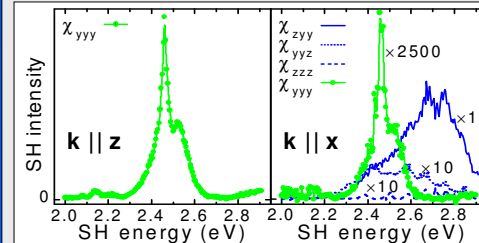
Lowest order non-zero contributions to SHG:

Zero order: Electric dipole (ED): $\chi^{ED}(P) = i, i, i, i$ and $\chi^{ED}(P \cdot \ell) = e_i$

First order: Magnetic dipole (MD): $\chi^{MD}(\ell) = m_i$

First order: Electric quadrupole (EQ): $\chi^{EQ}(\ell) = q_i, q_i, q_i$

	$S^{ED}(P)$	$S^{ED}(P \cdot \ell)$	$S^{MD}(\ell)$	$S^{EQ}(\ell)$
$k x$	S_y	$2i_j E_j E_z$	---	---
	S_z	$i_z E_z^2 + i_x E_x^2$	---	---
$k y$	S_x	$2i_j E_j E_z$	---	$-2q_j E_j E_z$
	S_z	$i_z E_z^2 + i_x E_x^2$	$m_i E_i^2$	$-q_j E_j^2$
$k z$	S_x	---	$-2m_j E_j E_x$	$-2q_j E_j E_x$
	S_y	---	$m_i (E_i^2 - E_j^2)$	$q_j (E_j^2 - E_i^2)$



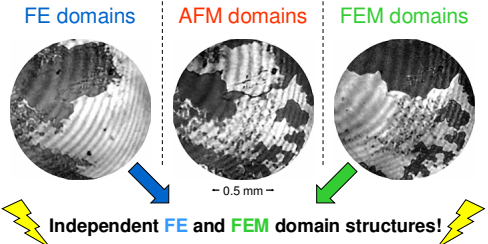
Identical magnetic spectra for $k || z$ and $k || x$

\Rightarrow bilinear coupling to $P \cdot \ell$.

First observation of ferroelectromagnetic (FEM) SHG!

Observation of ferroelectromagnetic domains

Simultaneous investigation of coexisting electric & magnetic domains in the same sample!



Independent FE and FEM domain structures!

Introduction of a FEM order parameter $F \equiv P \cdot \ell$

Clamping of order parameters:

\Rightarrow Ferroelectric (P) and ferroelectromagnetic (F) order parameters form independent domains.

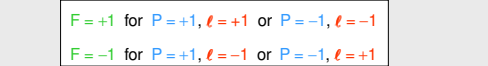
\Rightarrow Antiferromagnetic order parameter (ℓ) does not form independent domains \rightarrow AFM state is "unphysical"!

\Rightarrow Ferroelectromagnetic domains:

$$F = +1 \text{ for } P = +1, \ell = +1 \text{ or } P = -1, \ell = -1$$

$$F = -1 \text{ for } P = +1, \ell = -1 \text{ or } P = -1, \ell = +1$$

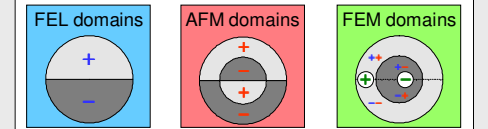
\Rightarrow Any reversal of the FE order parameter is clamped to a reversal of the AFM order parameter.



Identical magnetic spectra for $k || z$ and $k || x$

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First observation of ferroelectromagnetic (FEM) SHG!

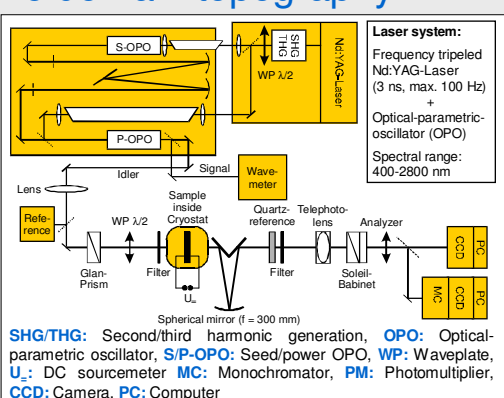
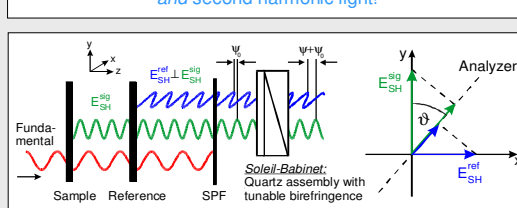


Nonlinear phase sensitive domain topography

Domain topography with external reference and tunable light source \Rightarrow problems:

- (i) Propagation effects due to distance between sample \leftrightarrow reference
- (ii) Inhomogeneous beam profile

Solution: Achromatic beam imaging of fundamental light and second harmonic light!



Microscopic theory

Origin of the SH signal:

- Mn^{3+} in trigonal bipyramidal field of oxygen ligands
- Local field distorted by ferroelectric ordering and spin-orbit interaction
- Exchange between adjacent Mn^{3+} ions

\Rightarrow Calculated spectral dependence of SH susceptibilities:

$$\chi_{yyy} \propto |P\ell| \left(\frac{1}{E_2 - 2h\omega} + \frac{\gamma}{E_1 - 2h\omega} \right) E_1 = 2.46 \text{ eV}, E_2 = 2.7 \text{ eV}, \gamma \ll 1,$$

$$\chi_{xyy} \propto |P\ell| \frac{v_x}{E_1 - 2h\omega} \quad v_x: \text{expectation value of crystal field induced by FEL ordering}$$

\Rightarrow Coupling of order parameters for the magnetically induced SHG!

Origin of the clamping of FE and AFM order parameters:

- Antiferromagnetic Mn-O-Mn superexchange
- Across FE domain wall decrease of the Mn-O(4)-Mn bond angle (3%)

\Rightarrow Exchange energy at a FE domain wall:

$$E = -(J_{(3)} + 2J_{(4)}) \pm (J'_{(3)} + 2J'_{(4)})$$

$$J_{(3,4)}, J'_{(3,4)} > 0: \text{Exchange integrals}$$

$$J'_{(3)} - J'_{(4)} < 0 \Rightarrow \text{Ferromagnetic coupling across FE wall!}$$

Alternatively: Magneto-elastic effects at FE domain wall.